THE PATH TOWARDS MANIFEST BACKGROUND INDEPENDENCE

Sabbir A Rahman[*]

Center for Theoretical Physics,
Laboratory of Nuclear Science
and Department of Physics
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139, U.S.A.

ABSTRACT

The set of string vertices is extended to include moduli spaces with genus and numbers of ordinary and special punctures ranging over all integral values $g, n, \bar{n} \geq 0$. It is argued that both the string background and the B-V delta operator should be associated with the vertex $\mathcal{B}_{0,1}^0$ corresponding to the once-punctured sphere. This leads naturally to the proposal that the manifestly background independent formulation of quantum closed string field theory is given by the sum of the completed set of string vertices $\mathcal{B} = \sum_{g,n,\bar{n}\geq 0} \mathcal{B}_{g,n}^{\bar{n}}$, satisfying the classical master equation $\{\mathcal{B}, \mathcal{B}\} = 0$.

1. **Introduction and Summary**

Since the proof by Sen and Zwiebach [1,2] of local conformal background independence of both classical and quantum closed string field theory, which stemmed from earlier work by Sen [3,4,5], much effort has been exerted towards constructing a formulation manifestly independent of the conformal background [6]. Similar efforts have also been made in the framework of open string field theory by Witten and Shatashvili [7,8,9,10,11]. Recent work on the dilaton theorem [12,13,14] by Belopolsky, Bergman and Zwiebach has shown that string backgrounds actually encode more data than conformal backgrounds, namely the vacuum expectation value of the dilaton field which governs the strength of the string coupling. This discovery called into question the previously held assumption that string field theories were built solely upon conformal field theories corresponding to classical solutions. This led to a search for more general string backgrounds, towards which strong progress has been made, notably in the recent work by Zwiebach [15], where it was sketched how classical closed string field theory might be built around non-conformal backgrounds without reference to any conformal field theory. Background independence was not quite yet manifest, as the string action was written as a function on the state space of an underlying two-dimensional quantum field theory which represented the tangent space to the space of backgrounds at that particular theory. However it did suggest that a manifestly background independent formulation may be within reach.

In two earlier papers [16,17] by the author, the string field theory operators ∂, $\mathcal{K}$ and $\mathcal{I}$ were expressed as inner derivations of the B-V algebra, and the recursion relations for the string vertices were shown to take the form of a quantum B-V master equation,

$$\tfrac{1}{2}\{\mathcal{B},\mathcal{B}\} + \Delta\mathcal{B} = 0 \,. \tag{1.1}$$

Here, $\mathcal{B} = \sum_{g,n,\bar{n}} \mathcal{B}_{g,n}^{\bar{n}}$ is the sum of the string vertices for all non-negative integers $g, n, \bar{n}$ except $g = 0, n + \bar{n} \leq 1$. The action was written in the simple form,

$$S = f(\mathcal{B}) \,. \tag{1.2}$$

In this elegant geometrised form, the only background dependence is contained in the string field $|\Psi\rangle$ and the fermionic state $|F\rangle$ required to define the functional f mapping the moduli spaces of decorated Riemann surfaces to functions on the space of fields and antifields.

In the present paper, which is slightly more speculative than the earlier works, We introduce the remaining negative-dimensional moduli spaces, $\mathcal{B}_{0,0}^0$, $\mathcal{B}_{0,1}^0$ and $\mathcal{B}_{0,0}^1$ in order to complete the set of string vertices, and incorporate them into the geometrised formulation of the theory. We argue in §2.1 that $\mathcal{B}_{0,0}^0$ and $\mathcal{B}_{0,0}^1$ decouple from the theory while the choice of string background is encoded into the space $\mathcal{B}_{0,1}^0$. The latter does not decouple, and to understand its antibracket action we propose in §2.2 a simple idea about its sewing properties which would suggest that the antibracket sewing of $\mathcal{B}_{0,1}^0$ should be identified with the action of the B-V delta operator. The set of string vertices then becomes a background independent algebraic structure containing complete information about the theory. This leads us directly to postulate in §2.3 the manifestly background formulation of quantum closed string field theory, where the 'geometrised' action is given by the sum of string vertices $\mathcal{B} = \sum_{g,n,\bar{n}\geq 0} \mathcal{B}_{g,n}^{\bar{n}}$, satisfying a classical master equation, $\{\mathcal{B}, \mathcal{B}\} = 0$.

2. Towards manifest background independence

In the geometrised formulation of the theory described by Eqns.(1.1)-(1.2) the theory is encoded elegantly into the set of string vertices satisfying a quantum master equation, as well as the string field $|\Psi\rangle$ and the state $|F\rangle$ defining the mapping f from surfaces to functions. To construct this functional we need a knowledge of the background which will determine the string field $|\Psi\rangle$ of ghost number two, and the fermionic state $|F\rangle$ of ghost number three. The set of string vertices are the background independent component of the formulation while the string fields are the background-dependent component. If we are somehow able to express this information in a background independent way, we will have achieved our ambition of attaining a manifest background independent formulation of the theory. A few suggestive observations will now lead us to postulate the way in which we expect this goal to be realised.

2.1. COMPLETING THE SET OF STRING VERTICES

The object $\mathcal{B}$ is the formal sum of moduli spaces of decorated Riemann surfaces $\mathcal{B}_{g,n}^{\bar{n}}$ for all positive values of $(g, n, \bar{n})$ with the exception of $\mathcal{B}_{0,0}^0$, $\mathcal{B}_{0,0}^1$ and $\mathcal{B}_{0,1}^0$. It would be satisfying if there could be some way of incorporating them into the theory and thereby completing the sum contained in $\mathcal{B}$.

Adding the spaces $\mathcal{B}_{0,0}^0$ and $\mathcal{B}_{0,0}^1$ should pose no problems. They are represented by an unpunctured sphere and a sphere with a single special puncture respectively, and their lack of

ordinary punctures implies that they do not couple in any way to the remainder of the theory, contributing at most a harmless constant to the action. The space $\mathcal{B}_{0,1}^0$ remains, and we shall discuss this object now.

We recall from §4 of [18] that the algebraic structure of the classical closed string field theory corresponds to a homotopy Lie algebra L_∞ defined by a set of multilinear graded commutative string products $m_n = [B_1, ..., B_n]_0$. For a conformal background, it was found that the product m_0 must vanish, $[\cdot]_0 = 0$, while for string theory around a nonconformal background it was associated to some non-vanishing ghost number three state $[\cdot]_0 = |\mathcal{F}\rangle \in \mathcal{H}$. Now, the product m_n is associated to the n-punctured vertex $\mathcal{B}_{0,n+1}^0$ with fields inserted at n of the punctures. So we expect the product m_0, which determines the string background, to be associated with a once-punctured sphere $\mathcal{B}_{0,1}^0$ with no insertions. Let us suppose that this is indeed the case, so that the background independent object $\mathcal{B}_{0,1}^0$ actually encodes the information about each particular string background.

Having introduced the spaces $\mathcal{B}_{0,0}^0$, $\mathcal{B}_{0,1}^0$ and $\mathcal{B}_{0,0}^1$, let us also assume that the master equation for the $\mathcal{B}$-spaces is still satisfied. We need to ask what the effect would be on the recursion relations. The spaces $\mathcal{B}_{0,0}^0$ and $\mathcal{B}_{0,0}^1$ have no sewable punctures, and can be safely ignored in this regard. However, $\mathcal{B}_{0,1}^0$ is not limited by this restriction. Let us then define the operator $\mathcal{U}$ as follows,

$$\mathcal{U}\mathcal{A} = \{\mathcal{B}_{0,1}^0, \mathcal{A}\}. \tag{2.1}$$

Existence of $\mathcal{B}_{0,1}^0$ then implies that there would be a contribution of $\mathcal{U}\mathcal{B}$ to our master equation. The vertex $\mathcal{B}_{0,1}^0$ has naive dimension -4, whereas the boundary operator ∂ is represented by the sewing of $\mathcal{B}_{0,2}^0$ which is of dimension -2. The presence of $\mathcal{U}\mathcal{B}$ would therefore signal the loss of the usual form of our usual recursion relations. If the contribution from $\mathcal{B}_{0,1}^0$ is nilpotent, one may be able to recover recursion relations of some kind, but they will not agree with the geometrical identity for the moduli spaces of Riemann surfaces. Furthermore we would also have to explain the existence (or vanishing) of new objects $\{\mathcal{B}_{0,1}^0, \mathcal{B}_{0,1}^0\}$, $\{\mathcal{B}_{0,1}^0, \mathcal{B}_{0,1}^1\}$ and $\{\mathcal{B}_{0,1}^0, \mathcal{B}_{0,2}^0\}$ which do not seem to have any interpretation in the theory as yet.

With a view to resolving these issues, let us consider the surface which $\mathcal{B}_{0,1}^0$ describes. It is a sphere with a single coordinate disk whose boundary corresponds to a Hilbert space. This is topologically equivalent to an infinite complex plane, the point at infinity representing the puncture and the boundary being some contour around the origin. This description is fine when there is no additional structure such as endowed by a metric. Suppose now that we try

to find a metric on the surface which satisfies the minimal area conditions [18]. In the first place, we note that attempting to put a metric on the sphere will always result in two singular points. One of these may conveniently be associated with the puncture, but the other one has no such identification. Adding to this the requirement that nontrivial closed loops have length $\geq 2\pi$, the geometrical description of $\mathcal{B}^0_{0,1}$ is as an infinite cylinder, completely foliated by saturating geodesics of length 2π [19], with a Hilbert space associated to the puncture at one end [Fig. 1]. This description will now prove useful as we attempt to identify the sewing of $\mathcal{B}^0_{0,1}$ with the B-V delta operation.
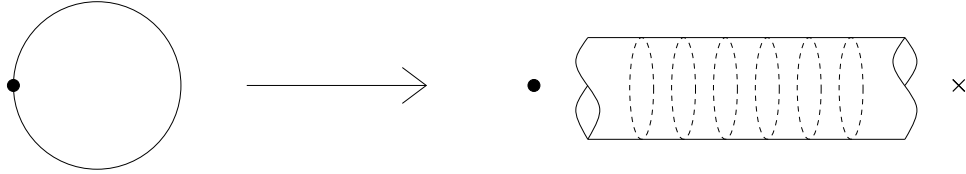

Figure 1. Equipping the once-punctured sphere $\mathcal{B}^0_{0,1}$ with a minimal area metric gives rise to an infinite cylinder with a Hilbert space associated to the punctured end.

2.2. THE SEWING OF $\mathcal{B}^0_{0,1}$ AND THE B-V DELTA OPERATOR

Let us consider the effect of sewing $\mathcal{B}^0_{0,1}$ in view of the above representation. Sewing it to another punctured surface will seemingly replace a puncture on that surface with an infinite tube. This would be fine if the tube ended at a puncture at the end of it with an associated Hilbert space, but this is not the case here. We propose that in order for this sewing to be well-defined, the boundless end of the cylinder must somehow lead to another Hilbert space on sewing, the only Hilbert spaces available being those (should they exist) which remain on the surface to which the space $\mathcal{B}^1_{0,1}$ is being sewn [Fig. 2]. What this proposition would suggest is that sewing $\mathcal{B}^1_{0,1}$ to a surface is equivalent to sewing together two punctures of that surface! In other words, we argue that the operator we denoted as $\mathcal{U}$ earlier is none other than the B-V delta operator 'Δ'. As an immediate corollary, the original quantum master equation for the complex $\mathcal{B}$ reduces to the classical master equation, $\{\mathcal{B}, \mathcal{B}\} = 0$.

It was shown at the end of [16] that the B-V delta operator cannot be an inner derivation of the B-V algebra, yet it may seem that we are attempting to do precisely that here by identifying it with the sewing of $\mathcal{B}^0_{0,1}$. However the sewing here does not coincide with antibracket sewing as is clear from the description we have given above, and so $\{\mathcal{B}^0_{0,1}, \cdot\}$ is not an inner derivation
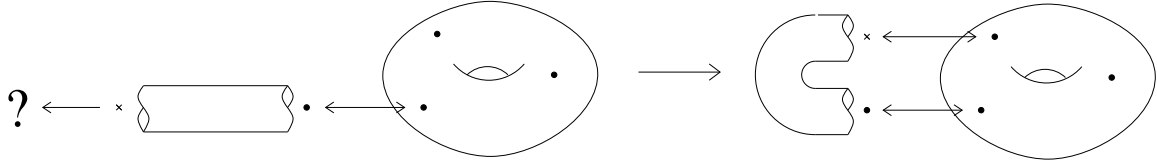
Figure 2. Sewing $\mathcal{B}^0_{0,1}$ to a surface effectively sews together two punctures of the surface.

on the algebra despite appearances. That the identities Eqns.(2.37)-(2.40) of [16] follow from an inner derivation interpretation will be considered as merely a happy coincidence. One might note that the objects $\{\mathcal{B}^0_{0,1}, \mathcal{B}^0_{0,1}\}$ and $\{\mathcal{B}^0_{0,1}, \mathcal{B}^1_{0,1}\}$ vanish from lack of sewable punctures, while $\{\mathcal{B}^0_{0,1}, \mathcal{B}^0_{0,2}\}$ gives the boundary of $\mathcal{B}^0_{1,0}$, as discussed in [17].

2.3. Manifest background independent formulation

If we suppose that the identifications we have suggested above are correct, then it is no longer necessary to retain explicitly the action S, as all the information required to encode the theory is already contained in the background independent algebraic structure defined by the complete set of string vertices. Indeed our final 'manifest background independent' formulation of the full quantum closed string field theory would be given by the following 'geometrical master action',

$$\mathcal{B} \,, \tag{2.2}$$

(where $\mathcal{B} = \sum_{g,n,\bar{n}\geq 0} \mathcal{B}^{\bar{n}}_{g,n}$), satisfying the 'geometrical classical master equation',

$$\{\mathcal{B}, \mathcal{B}\} = 0 \,. \tag{2.3}$$

We shall conclude our analysis at this point.

3. Conclusion

If the arguments which have been been presented in this chapter can be made rigorous, we will have succeeded in finding a fundamental geometrical representation of string field theory which underlies the usual formulation. Furthermore, the unexpected appearance of the Batalin-Vilkovisky master equation, which is satisfied by all gauge field theories strongly suggests that this is a phenomenon which is not restricted to string theory, and may be of much more general applicability, where our string vertices $\mathcal{B}$ would be replaced by more general algebraic objects.

There are still many gaps in our understanding which would need to be filled before we can be satisfied that we have found a manifestly background independent formulation of string field theory. The arguments we have brought forward merely suggest that our identifications are consistent, and certainly do not represent proof of correctness. Besides this, our work is largely dependent on the postulates of [15], some of which have yet to be firmly established. As particular examples, we do not yet have explicit expressions for the BRST operator Q, the fermionic state $|F\rangle$, the one-form $F^{[1]}$ or the two-form $F^{[2]}$, and are therefore unable to explicitly construct the action or extract physical states for general backgrounds. Also, some of the issues which remained open at the end of [15] still remain to be tackled. There are doubtless other difficulties which we have not mentioned or of which we are as yet unaware.

Having said this, it is of interest to generalise our ideas to the case of open-closed string theory [20], and superstring field theories as formulated in the language of [21], in the hope of eventually clarifying important issues such as the nature of the space of string backgrounds and the existence of dualities. If such a program is successful, one would expect that the results would also be of direct relevance to field theory as a whole. Of course much more work needs to be done before such ideas can be properly established but we hope nevertheless that the ideas suggested in this paper will lead to some solid progress in our basic understanding of string theory.

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